

I. Introduction

As mentioned at the beginning of the course, there are two widely used models of rational choice: (a) the net benefit maximizing model and (b) the utility maximizing model. In both cases individuals are assumed to have goals that can be mapped into a single index of well being, either net benefits or utility. In both cases, the relationship between these numbers and quantities of goods and services can be used to develop a logical structure for market and other human activities.

The utility maximizing model has two advantages as a model of rational choice: (a) it does not require the ability to map from activities or quantities into dollar amounts, and (b) it can be easily generalized to deal with choices involving more than two dimensions and to non-market activities.

The concept of “utility” or “satisfaction” or “happiness” was worked out by a group of social scientists and intellectuals during the nineteenth century, who were interested in improving public policy. “Utilitarians” argued that a good policy should maximize social welfare defined as the sum of the utilities of all the persons in society (Bentham, Mill, etc.).

In addition to being interested in normative theory, a subset of the persons in this group were interested in narrow questions about how prices came about and why patterns of exchange and organization look as they do. Towards the end of the nineteenth century, the idea of diminishing marginal utility was introduced more or less simultaneously by three

economic theorists in three different countries (William Jevons, Karl Menger, and Leon Walras). They each had slightly different conceptions of this principle, but agreed that successive units of a good increase utility by less and less.

This idea was used to explain the diamond-water paradox. Water is extremely valuable--life is impossible without it--whereas diamonds are of trivial practical value. Yet diamonds are more valuable than water, as evaluated by market prices. Proponents of the theory of diminishing marginal utility argued that prices are determined by marginal utility rather than total utility (as in our marginal benefit and marginal cost based derivations of demand and supply). Water was worth less at the margin (because we have so much of it) than diamonds (because they are so rare).

This one dimensional use of utility was generalized to multiple dimensions through the development of the idea of a utility function, $U = u(a, b, c, d, \dots)$ Where U is utility and a, b, c, d, \dots are quantities of different goods and services. This allowed more complex choices and decision environments to be examined.

In intermediate microeconomics we will focus on cases in which there are two goods in the utility function. This is partly because we use geometry (two-dimensional diagrams) to represent utility maximizing choices and partly because most of what can be deduced from broader n -good models can be deduced from 2-good models.

The theory of demand and supply was initially worked out independently of marginal utility theory in a manner analogous to what we used in the first part of the course. However, additional insights about complementary goods, substitute goods, and income can be obtained using the two-dimensional models that we will focus on for the next few weeks.

Historically, it turned out that it was only a small step from the idea of utility functions to the concept of indifference curves and a more complete geometric way of thinking about both demand and marginal utility simultaneously--although as it turns out marginal utility became relatively unimportant in the indifference curve based geometric framework.

II. Geometric representations of Utility Functions

It is possible to represent a 2-good utility function [$U = u(a, b)$] in two dimensions using a technique similar to those used by map makers when they try to represent a mountain range on a flat piece of paper. A **topological map** draws contour lines which represent places where the height above sea level are the same. A map of a utility function draws contour lines that represent equal utility levels for different combinations of goods and services (different “bundles”).

These contour lines are called “indifferent curves” and represent all the combinations of the two goods in the diagram of interest that generate the same level of utility.

In the standard textbook diagram, indifference curves are downward sloping c-shaped curves, and utility increases as one moves out to the “north east.” “More is always better” in textbook diagrams.

To represent a choice setting for a given person requires both a representation of his or her **utility function** (a series of indifference curves) and an **opportunity set**.

The indifference curves show the total benefits associated with different combinations of goods (in “utils” rather than in dollars).

The opportunity set represents what is possible for the person given his or her circumstances.

In most economic illustration, the opportunity set is **determined by personal wealth and market prices**.

A rational utility maximizing individual will always choose best (utility maximizing) option available to him (or her).

In the “textbook” consumer choice setting, a person’s opportunity set is the set of all combinations of goods that their current income or wealth allows them to purchase. If Al has a hundred dollars to spend on goods A and B, and good a costs \$20 each and good B costs \$10 each, she can purchase any combination (Q_A, Q_B) such that $\$100 = 20Q_A + 10Q_B$. More generally, if Al has W dollars to spend on goods A and B, she can purchase any combination of the goods that costs less than W, or any Q_A and Q_B such that $W \geq P_A Q_A + P_B Q_B$. This algebraic way of representing a what AL can buy is called a **budget constraint**.

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A 2-good budget set, budget constraint, or **budget line** can be illustrated with a diagram, as done below in figure 1. The highest utility level that can be realized given a budget set is the consumer's choice, his or her most preferred "bundle" that is feasible. This normally occurs at a place where an indifference curve is tangent to a budget line, as developed below.

III. Optimizing given Budget and other Constraints

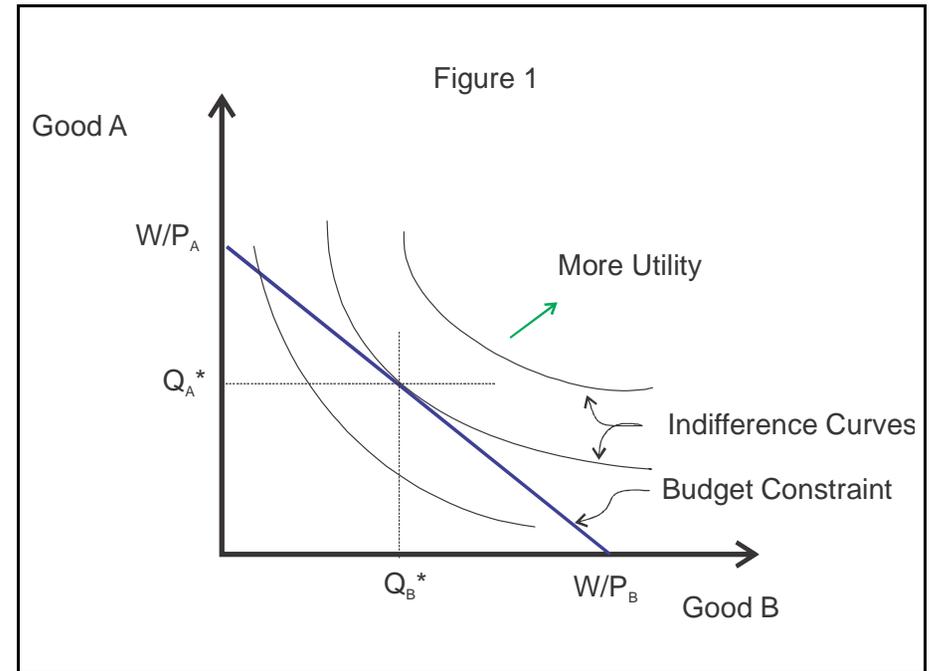
A. We now have the basic ideas (concepts) necessary to develop the utility-maximizing model of rational choice by consumers.

- We represent a person's goals using indifference curves (or a utility function).
- We represent their opportunities in market settings (where consumers are price takers facing well-known prices) using a budget constraint.

B. A rational consumer will choose the combination of goods that maximizes his or her utility, which is the combination of goods that he or she can afford that places him or her on their highest feasible indifference curve.

- The "highest" indifference curve is the indifference curve that represents the highest utility level.
- Figure 1 depicts a typical consumer's set of indifference curves and budget line (set).
- In the case usually drawn, the highest indifference curve that can be reached is at place where an indifference curve is tangent to the individual's budget constraint (budget line).

- [However, **remember that the tangency condition is just a rule of thumb.** What is always chosen by a rational consumer is the feasible combination of goods and services that maximize utility (given a particular opportunity set).]



C. In figure 1, note that just three indifference curves are drawn.

- In reality there are an infinite series of these curves.
- However, three or four are normally sufficient to give one a sense of what a utility function (mountain) looks like.
- "More is better" over the range of interest, because more of either or both goods always produces more utility.

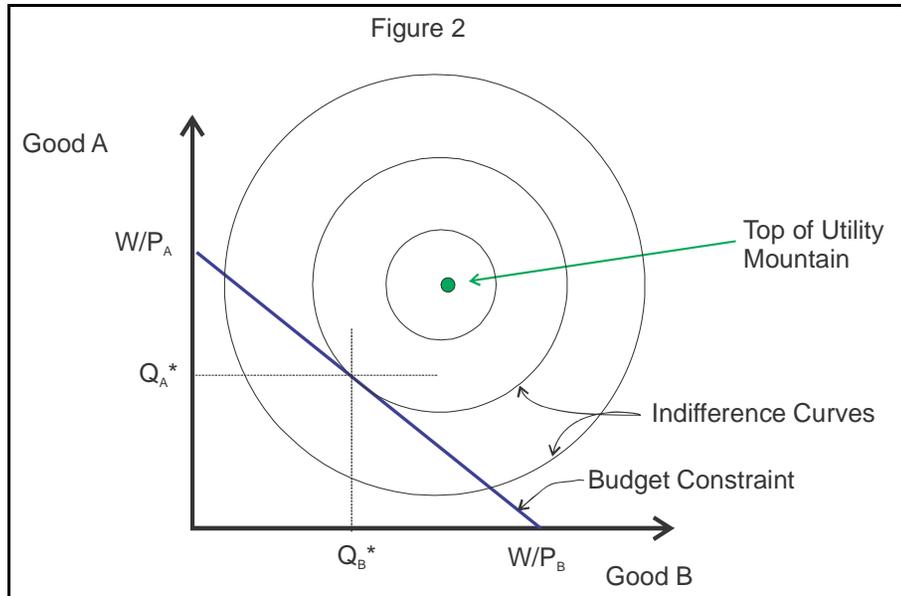
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- More of either or both goods always puts AI on a higher indifference curve.
- D. Notice that the outer edge of the budget set is a straight line that goes from W/P_A to W/P_B .
- These two extreme points have an intuitive meaning.
 - If AI spent all of her money on good A, she could buy W/P_A units. If AI spent all of her money on B, she could buy W/P_B units.
 - That the budget constraint is a straight line is implied by the equation, $W = P_A Q_A + P_B Q_B$.
- E. Normally, AI can purchase any quantity inside the triangle formed by her budget line and the good A and good B axes, but since A and B are usually both goods (more is better), only the outer edge of the budget set is important.
- (The outer edge of a budget set is usually called the “budget line” or the “budget constraint.”)
 - The slope of the budget line turns out to be $-P_B/P_A$ in the diagram if “A” is interpreted as the “Y” axis. [This ratio is sometimes referred to as the **relative price** of B in terms of A.]
 - (As an exercise take the above equation for a budget constraint and put it in the conventional form of $Y = a + bX$. Note that Y is Q_A and X is Q_B in this diagram.)
- F. If you look at AI’s choice, you should be able to see that **highest indifference curve** that can be reached is one that just touches her budget line. Such an indifference curve is said to be **tangent** to the budget line.
- It is the combination labeled Q_A^* , Q_B^* in figure 1.
- G. This model of the rational consumer allows us to characterize the choice over various combination of goods, given any opportunity set.
- This allows us to get a more complete picture of how individuals adjust to changes in prices and income--and any other factors that may change their opportunity sets.
 - It also allows us to analyze a broad range of choice settings, types of individuals, and changes in circumstances.
 - The axes can be time, effort, goods, services, etc.
 - All we need are indifference curves and opportunity sets to apply this model.
- H. Because you can analyze settings that are more complex than possible using the net benefit approach, the “constrained optimization” model of rationality is the dominant one in Economics, Game Theory, and Rational Choice Political Science theory courses.
- This does not, however, mean that the net benefit maximizing model with its assorted area tools is never used.
 - In fact, in most upper level “applied micro” courses, they are used more frequently than the utility-maximizing model, which is why we spent half the course on them.
 - This is true even of many graduate courses in public policy analysis.
- I. The basic consumer model can be extended to take account of different kinds of persons (e.g. persons with different tastes) and/or to persons who face different constraints.
- Figure 2 shows a person with “odd” preferences

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- This person has a utility function with a “bliss point,” a top, just as ordinary mountains do.
- More is not always better for this person. He/She can become satiated with both goods.
- Such a person will often, but not always be influenced by changes in their wealth or prices.
- [Illustrate by shifting the budget line and/or explain why his/her choices are sometimes but not always affected by an increase in wealth.]



J. The geometry of maximizing utility subject to a budget constraint is very flexible and geometrically very different from that of the net benefit maximizing model, but it is really fundamentally similar.

- In both cases, rational individuals are assumed to maximize something (net benefits or utility) that is only partly under their

control, because they have to pay market prices for the goods of interest. In both cases there are “conventional” and odd cases that can be analyzed. In both cases, a theory of demand can be worked out based on the implications of rational choices in settings where consumers are price takers.

IV. Deriving a Demand Curve using Indifference Curves and a Series of Budget Constraints

A. The meaning and logic of demand curves **are not changed** when we shift from the net-benefit to the utility maximizing representation of rational choice.

- In both cases, a demand curve tells us how much of a particular good a person will purchase at various prices.

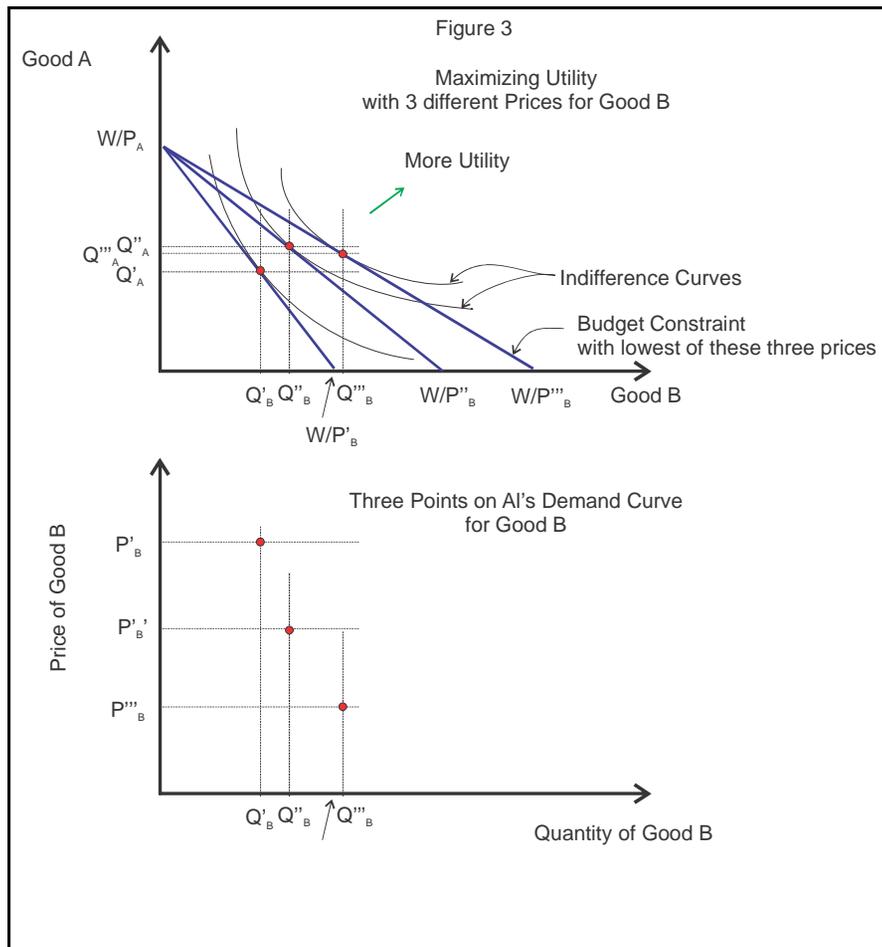
B. The basic steps that one takes to derive a demand curve do not really change, although the geometry does.

- (i) Choose a price,
- (ii) Find the implied budget constraint,
- (iii) Determine the feasible combination of goods with the highest utility,
- (iv) Plot the price and quantity of interest on a separate diagram.
- (v) This process is repeated with a series of prices to trace out the entire demand curve. [Pick a new price and go to (ii)]
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- This process is illustrated in Figure 3 for three prices.

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- Note that when ever a price changes the budget constraint “rotates.” The new price changes the slope of the budget line. It also changes the intersection of the budget line with the axis of the good with the new price.



- As prices fall for the goods along the horizontal axis the budget constraints become flatter.

- Nonetheless, the highest indifference curve that can be reached is always (in the normal textbook case) a place where an indifference curve is tangent to the budget line of interest.
- Note that the **quantity of A demanded is also normally** affected by changes in **price of B** and changes in income or wealth. This helps explain why the demand for a good is affected by the prices of other goods: substitutes and complements, and why demand tends to rise as income increases.
- [As an exercise, draw an odd set of indifference curves and derive a demand curve from them.]

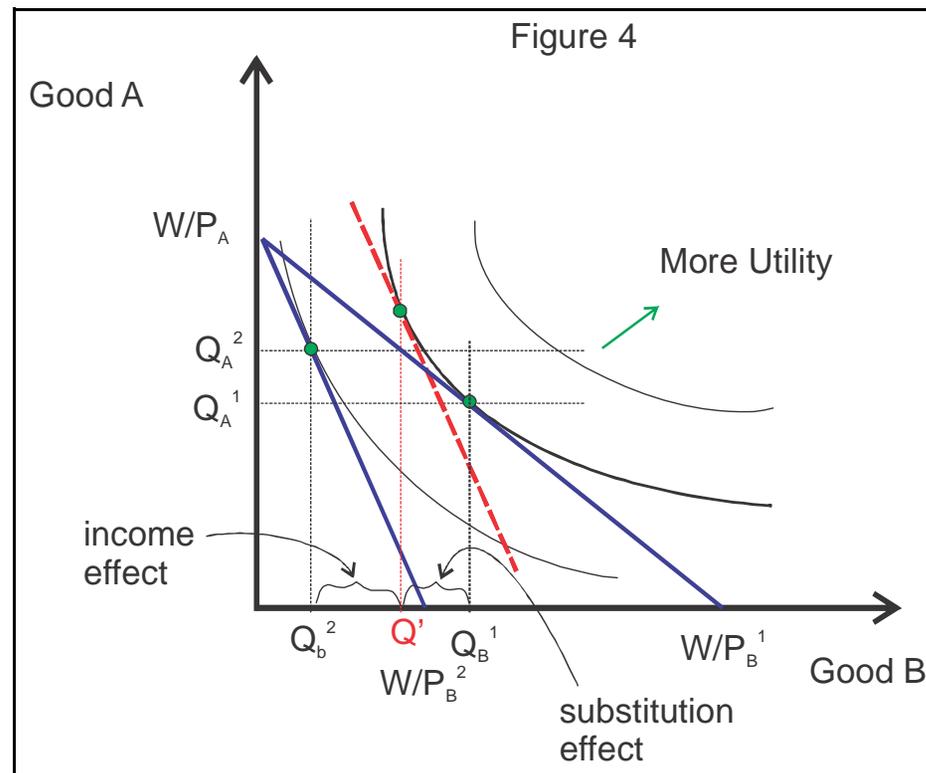
V. Income and Substitution Effects

- In the net-benefit maximizing framework, we were able to show that demand curves **always** slope downward. That is to say, all the points on an individual's MB curve that make their way to his or her demand curve were from the downward sloping parts of the MB curve.
- This is **usually the case** for demand curves derived from the utility maximizing model, but **not always**.
 - To make sense of this surprising result, economists have distinguished between **income and substitution effects**.
 - When indifference curves slope downwards and are C-shaped, the substitution effect of an increase in prices is always negative. One always tend to reduce purchases of the good with the new higher price and substitute the other good for it.
 - However, real income falls because the budget set shrinks as prices increase.

- The implied “income effect” may be negative or positive according to whether the good is a “normal or superior” good or a “inferior” good.
- **DEF: Inferior goods** are goods that one purchases more of when one has less money, wealth, or income.
 - As prices rise, one become poorer (one’s opportunity set shrinks) and one **may purchase more inferior goods**.
 - Inferior goods thus have positive income effects that partly or entirely offset the substitution effect of a price increase.
- A **Giffen good** is an extremely inferior good. Its income effect is so strong that it overwhelms the substitution effect and generates an upward sloping demand curve—at least in theory.
 - [It is often claimed that no Giffen goods have ever been found. If so, what is the point of working so hard to characterize income and substitution effects?]

C. Geometry of Income and Substitution Effects.

- Draw a standard budget constraint with an indifference curve tangent to that budget constraint.
- Raise the price of one of the goods. (I will raise the price of the good on the horizontal axis, but either will work.)
- Note that the higher price changes the slope of the budget line and implies a new utility maximizing combination of goods.
- Draw in an imaginary budget line **parallel to the new budget line**, but tangent to the original indifference curve (the indifference curve that was tangent to the first budget line).
- Note the new optimal bundle implied.



- The **substitution effect** is the change in the quantity of the good whose price has increased from the original bundle to the imaginary one on the red dotted line. [It is the part of the change in quantity purchased generated by the change in relative price.]
- The **income effect** is the change from that quantity to the one on the actual budget line chosen after the price increase. [It is the part of the change in quantity generated by the shrinking of the budget set.]

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- [There is not much intuition here, so just memorize this method. Its one of the core concepts of a “normal” intermediate micro course, so it is very likely to turn up on the final.]

D. One can also use indifference curves and budget constraints to represent how patterns of consumption will change when wealth or income changes, by changing W rather than a price.

- The result is an **income-consumption path**
- The income-consumption path can be used to derive an Engels curve, which describes how income affects the quantities of a good that a consumer purchases of a particular good as his or her income changes.
- To derive an Engels curve, vary W and observe how the quantity of a good varies. Plot W and the quantity of one of the goods of interest.
- [An Engels curve will be derived in class, but try to derive an income consumption path, and plot out an Engels curve as an exercise before class.]

VI. The Edgeworth Box: Equilibrium in a Barter Economy

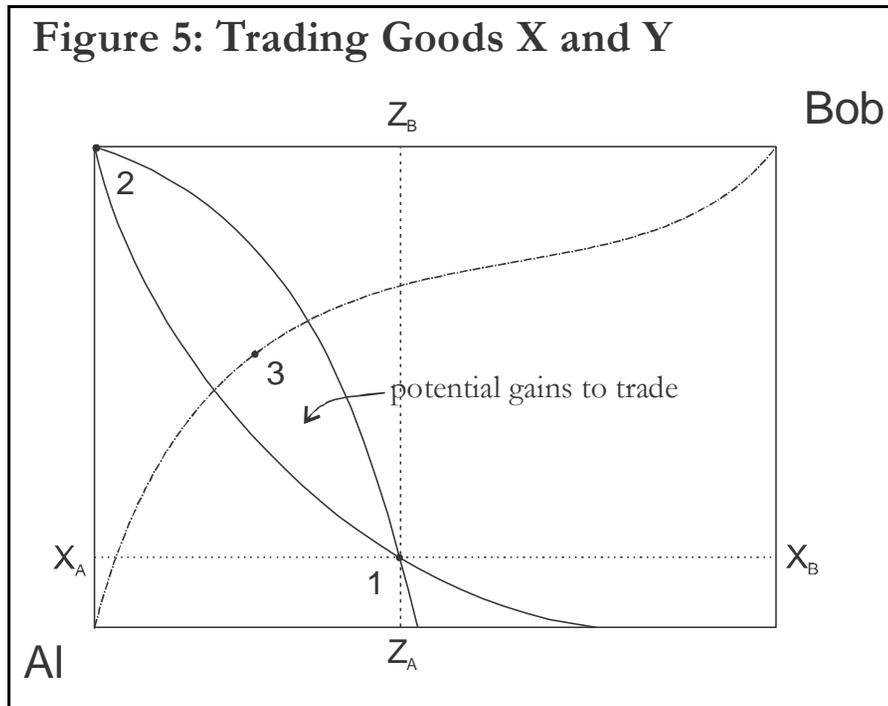
A. One of the extremely clever things that one can do with indifference curves and budget constraints was worked out by an economist named Edgeworth in the late nineteenth century.

- Francis Edgeworth found that by drawing two indifference curve diagrams and combining them, he could illustrate (i) why gains to trade exist and (ii) what a “general equilibrium” price looked like in a barter economy (an economy where goods are directly traded for one another).

- Edgeworth’s diagram shows that for most points in the box, there are unrealized gains to trade.
- It also shows how a single relative price can simultaneously clear two markets.
- (Walra later extended this idea to represent a “general equilibrium” in which all markets for all goods are simultaneously cleared by “vector” of prices for each good sold.)

B. Trade can make both persons better off from most places (initial endowments) inside an Edgeworth box.

- An initial endowment is a point in the Edgeworth that represents two persons ownership (holdings) of the two goods.
- The region in which gains to trade exist can be found by drawing an indifference curve for each person through the **initial endowment** (the point in the box representing the initial division of the resources in this 2-person society).
- The football shaped area between the indifference curves is the region in which gains to trade exist. It is the set of points that are Pareto superior to the initial endowment.
- Note the similarity of the idea of a Pareto superior move and mutual gains from trade in this “trading setting.”
- Moves from any point of interest (such as the one’s labelled 2 or 1) can be assessed to determine whether a move is Pareto superior or not.
- Mutual gains from trade exist for most of the points in the box.



- The points where no gains to trade exist are those where one person's indifference curves are tangent to the others. [Explain why.]
- The collection of all such points is called the **contract curve**.
- Note that when trades do take place, trading is predicted to continue until a point on the contract curve is reached.
- All the points on the contract curve are Pareto optimal. At those points no (feasible) Pareto superior moves exist.

- Given an initial endowment, the “football” of gains to trade represent all the potential Pareto superior moves from that endowment.
- The area of mutual **gains to trade** was later called a **Pareto set**.

C. Pareto norms and the Edgeworth Box

- A move from X to Y is a Pareto superior move if and only if at least one person prefers Y to X and no one prefers X to Y
- A social state is Pareto optimal if and only if no Pareto superior moves exist.
- (Vilfredo Pareto was an Italian economist. Both Edgeworth and Pareto overlapped with Jevons, Menger, and Walra, who launched the “marginal revolution,” but Pareto was from the next generation of economists.)
- The Pareto set for a given initial endowment is all the distributions of the goods (points in the Edgeworth box) that make one person better off and no one worse off.
- State of the world 3 in figure 5 is said to be **Pareto Superior** to state 1, because at least one person prefers 3 to 1 and no one prefers 1 to 3.
- State of the world 3 is said to be **Pareto Optimal** because there are no (feasible) Pareto Superior Moves at 3.
- The **contract curve** is the set of Pareto Optimal states in an Edgeworth Box. (It is made up of the points where the indifference curves of Al and Bob are tangent to one another.)
- (As an exercise, demonstrate that position 2 is not Pareto superior to position 1.)

VII. Production Functions

A. A similar geometry can be used to analyze a firm's choice of production methods.

- Two dimensional choice models can be used to analyzing how a firm adjusts its input mix as it expands production.
- It can also be used to analyze how changes in input prices affect the inputs used in production and the total cost curve (and thereby the marginal cost curve) of a firm.

B. We have seen that the net-benefit maximizing framework explains the output decisions of firms quite well.

- However, it does not shed much light on how a firm will produce its outputs.
- We know from that approach that a rational firm will minimize the total cost of each unit of output produced, because that is necessary if profits are to be maximized.
- However, we do not really have a clear sense of how inputs are combined to produce outputs.

C. The idea of a production function can be used to explore how output Q may be produced from different combinations of inputs, x_1, x_2, x_3, \dots , with $Q = f(x_1, x_2, x_3, \dots)$

- Output is the aim of production. It is what one plans to sell.
- Inputs are all the labor, machines, and materials used to produce the output.
- The firm will attempt to use the least cost combination inputs to produce its output.

- This, as it turns out, can be represented using diagrams similar to those used for the consumer's utility maximizing choice.
- One can represent a 2-input production function on a diagram using topological maps of a production function and a cost function.

D. Isoquant and Isocost lines

- Isoquant curves represent all combination of inputs that can be used to produce a particular level of output (without waste).
 - They generally are assumed to be C-shaped, although other shapes are possible
- Isocost curves represent all combinations of inputs that can be purchased for a given amount of money (all combinations that in total cost the same).
 - These curves have the same general shape as budget lines (in competitive markets), whenever inputs are purchased in competitive markets (e.g. at given prices). [Explain why.]

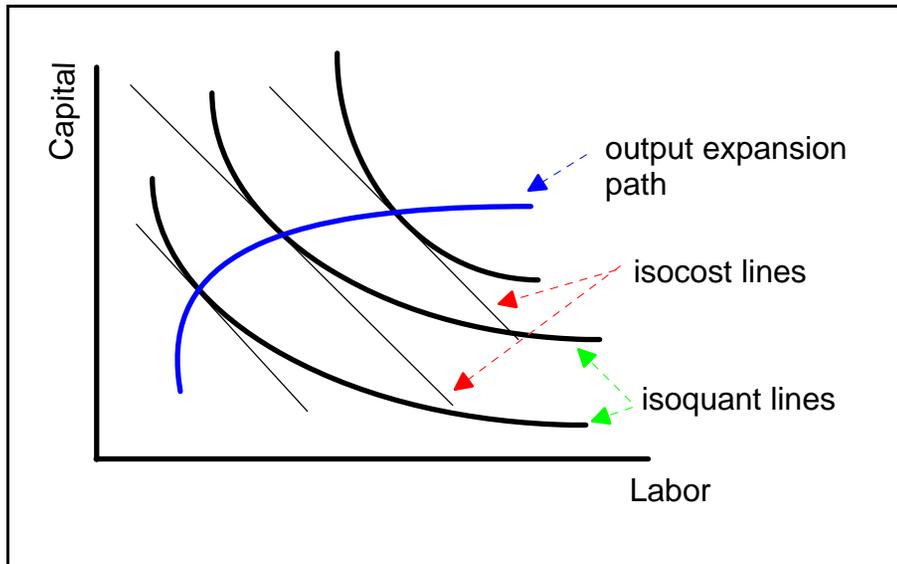
VIII. Choosing the Best Way to Produce: the Output Expansion Path

A. Production Expansion Paths

- The firms will want to maximize the output that can be produced for a given cost.
- The combination of inputs that minimizes the cost of a given output occur where an **isocost line is tangent to an isoquant** curve.

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- The results characterize a firm's "output expansion path." It shows how a firm will change its input level and mix as production expands.
- Note that exactly the **same output expansion curve** can also be derived by maximising the output achievable at a given cost, for a series of isocost curves.



B. Output Expansion Paths and Total Cost Functions

- The **output expansion path** is the series of input combinations that a firm will adopt as it increases output.
- These occur at the tangencies of the isocost and isoquant curves.
- This series of tangencies **also** indirectly describes the **firm's total cost function**, the minimum cost of every level of output.

- If you plot the output (Q) on the isoquant line and cost represented by the isocost line at points where the two curves are tangent, you will have plotted the firm's total cost function for given input prices.
- And, of course, once you know a firm's total cost function you can determine its marginal cost function and thereby its supply curve using the method worked out earlier in the course.
 - Recall that marginal cost is the change in total cost for a one unit change in output, it is the slope of the total cost curve.
 - Note that this implies that total costs (and marginal costs) will change when input prices change.
- (Note to student: most of the material in this lecture is easier to develop with calculus, but you are expected to know this geometry after a "normal" intermediate micro course, so expect a question about isocost and isoquant curves on the final.)

C. Effects of changes in Input Prices

- In addition to providing a richer foundation for cost curves, isocost and isoquant based analysis can also be used to show how changes in input prices will affect a firm's best method of producing its output(s).
- A change in the price of either (or both) inputs changes the slope of the entire series of isocost lines.
- The new series of isocost lines implies a new series of tangencies and so a new production expansion path and a new total cost and marginal cost curve.
- [Derive an output expansion path by drawing three isocost lines. Next, draw three new isocost lines with new slopes (because of input price changes). Note that the production expansion path is

affected by the change in relative prices (slope of the isocost curves.)

D. Note that this method shows how and why a firm's demand for inputs is affected by input prices.

- Firms will substitute away from the input with a higher price at every output level.
- Thus, in general, the demand for inputs is downward sloping.
- **(Note that this is very similar to our results using marginal revenue product curves in the first half of the course.** However, the isoquant-isocost based analysis show more of the adjustments being made by firms as they increase outputs or adjust to changes in market prices for inputs.)

IX. Technological Innovation and Production

A. The technology of production has been assumed constant in the above diagrams.

- However, technological change can also be modeled in isocost-isoquant diagrams.
- Technological shifts changes in the shapes of a firm's isoquants.
- Technological advance in production methods implies that **the new isoquants are higher** than the old ones for at least some combinations of inputs. Given the new technology, one can produce the same output with fewer inputs, or more output from the old input combinations.
- [In the net benefit framework, a technological improvement can be represented as an increase in the marginal product of at least

one input, which implied a reduction in the firm's MC of production. Again the results are similar, but additional details are provided by the isoquant-isocost framework.]

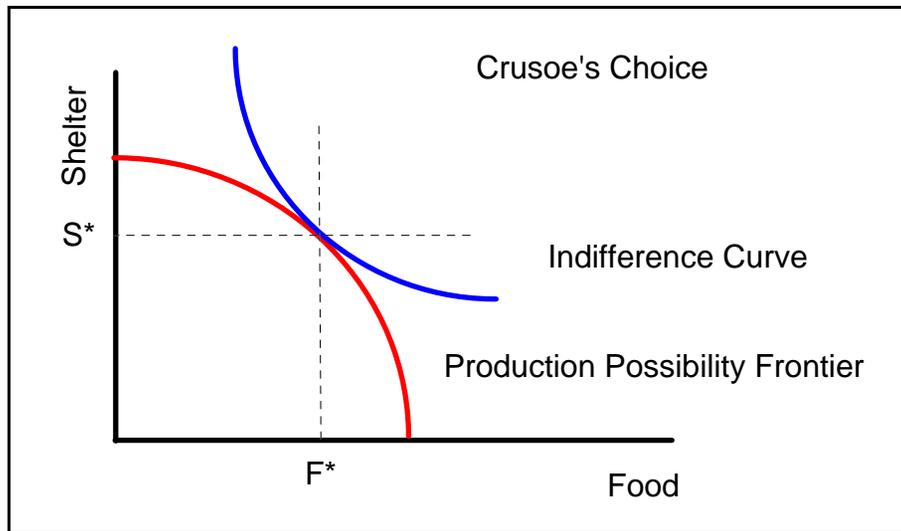
B. Production possibility frontiers

- Its interesting to note that we can combine production and consumption decisions into a single diagram.
 - For example, a self-sufficient farmer choose the mix of outputs to produce based in part on production technology and inputs and in part on the value that he/she places on alternative outputs.
 - We can use a "production possibility frontier" to characterize the range of outputs that can be produced from a self sufficient farmer's available inputs.
 - Given a finite set of inputs, one can, in principle, plot all the possible combinations of outputs that can be produced.
 - This "constraint" over alternative outputs is partly caused by the stock of inputs and partly by technology.
 - **It is called the production possibility frontier.**
 - Production possibility frontiers can be used to think about personal choices, firm choices, and national choices.
- C. For example, Robinson Crusoe's problem when shipwrecked on an island was deciding what to produce given his labor, his knowledge of production techniques, and the resources of "his" island.
- His resources implied a multidimensional production possibility frontier.
 - His preferences (utility function) allowed him to prioritize certain kinds of products for production.

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- His choice can be illustrated in two dimensions if we focus on two of his outputs, say food and shelter, or fish and coconuts.
- [Draw a production possibility frontier and series of indifference curves and show Crusoe's utility maximizing combination of fish and coconuts.]



X. The geometry of market equilibrium with indifference curves and isoquants

A. Notice, that you can also use the new tools to represent a market equilibrium, and include several possible diagrams to represent the effects of market prices on individual consumers purchases and firm output decisions, and also on how inputs prices affect production methods.

- Both net-benefit and utility-maximizing representations of consumers and firms are possible.

- In general, both sets of tools show how prices affect decisions throughout markets.
- Prices thereby coordinate the choices (and behavior) of a huge number of consumers and firms in a huge number of markets.

B. Important Exercises: Linking Market Outcomes, Consumer Choices, and Production Choices

- As an exercise draw two output (final goods) markets in equilibrium and note how that equilibrium affects a typical consumer's decision to purchase goods from these markets.
- As an exercise draw two input markets in equilibrium and show how the equilibrium affects a typical firm's decision to produce its product using isocost and isoquant curves.
- Now show how a firm's decision to employ inputs in the above case depend on its profit maximizing output level given the costs implied by the above diagram, but also by marginal revenues as determined in the final goods market.
 - Note that you'll need a net benefit maximizing diagram of the firm's output decision to determine which isoquant the firm decides to be on.
 - For the theory of the firm, both net benefit and the two dimensional production models can be simultaneously used.
 - (In the case of the consumer choice models, one normally uses either the net benefit model or the utility maximizing model, but not both.)
- In contemporary world markets, prices affect the decisions of thousands of firms and millions of input providers and consumers.

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- [Draw demand and supply curves, and individual consumer and firm curves based on the two dimensional models of this handout.]
- [Compare these with their counterparts from the previous handout.]
- [Note that both sets of diagrams have similar implications, but the ones based on this handout (3) are a bit more detailed.]